COMMENTS

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Comment on "Critical dimensionalities of phase transitions on fractals"

Raffaella Burioni*
Dipartimento di Fisica, Università di Roma "La Sapienza," Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Piazzale Aldo Moro 1, 00185 Roma, Italy

Davide Cassi†
Dipartimento di Fisica, Università di Parma, INFM-CISM, Unità di Parma, Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Parma, Viale delle Scienze, 43100 Parma, Italy
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In a recent paper [Phys. Rev. E **49**, 99 (1994)], Shi and Gong have proposed a necessary and sufficient condition for spontaneous symmetry breaking on fractals. In this Comment, we show that this condition does not hold, presenting some counterexamples and critically analyzing their proof.

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In a recent paper Shi and Gong [1] analyzed the problem of phase transition on fractals, giving by heuristic arguments a sufficient and necessary condition for spontaneous symmetry breaking at finite temperature. Their claims are the following.

(1) On finite ramified fractals (FRF's) discrete symmetries are spontaneously broken at $T>0$ if and only if $\tilde{d}\geq 2$, where $\tilde{d}$ is the spectral dimension of the fractal structure.

(2) On FRF's continuous symmetries are spontaneously broken at $T>0$ if and only if $d_F \geq d_W+1$, where d_F is the fractal dimension and d_W is the random walk dimension.

(3) Both discrete and continuous symmetries are broken at $T>0$ on infinitely ramified fractals (IRF's).

In the following we will show that (1) the condition concerning discrete symmetries on FRF's is not sufficient; (2) the condition concerning continuous symmetries on FRF's is not sufficient; and (3) the condition concerning IRF's is not sufficient for continuous symmetries.

In order to prove our statements, we refer to two counterexamples, namely Sierpinski carpets (SC's) and "Nice Trees of dimension D" (NTD).

The first counterexample concerns a class of fractal trees called NTD. For these fractal structures $\tilde{d}=d_F=1+\ln k/\ln 2$, where k is as arbitrary positive integer [2]. This relation implies that $d_W=2d_F/\tilde{d}=2$ for every k. Then the condition $d_F \geq d_W+1$ is fulfilled for all NTD with $k\geq 4$, and the condition $\tilde{d}\geq 2$ holds for $k\geq 2$. However, spontaneous symmetry breaking at finite temperature cannot occur on tree structures with finite d_F both for discrete and continuous symmetries [3]. So NTD with $k\geq 2$ represent a counterexample for the sufficient condition in the case of discrete symmetries (1), and NTD with $k\geq 4$ represent a counterexample for the sufficient condition in the case of continuous symmetries (2).

In addition, it is known that random walks on a SC are recursive since $\tilde{d}<2$ for this structure [4–6]. Then, from the generalized Mermin-Wagner theorem [7], it follows that continuous symmetries are not spontaneously broken on a SC. This proves our statement (3).

In order to understand why the heuristic arguments given by the authors do not apply to the counterexamples, we point out that their hypothesis that the surface of a domain of linear size L can be covered by a random walk does not hold in general since (see, e.g., NTD) the domain surface can be a disconnected graph.

Moreover, the assumption that in the infinite ramified case the energy required to create a domain wall is infinite is not correct. Indeed, the definition of infinite ramification only applies to the continuum limit of a fractal: a finite domain in the continuum limit corresponds to an infinite domain on the discrete structure.

*Electronic address: BURIONI@ROMA1.INFN.IT
†Electronic address: CASSI@VAXPR.PR.INFN.IT
37993::CASSI

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